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Parameterisation of eddies in coarse resolution models

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Abstract. Some requirements on eddy parameterisations are discussed, especially the implications of expressing them in terms of the quasi-Stokes velocity and the modified mean, rather than Eulerian mean, density. The difference between the two means is second-order in perturbation amplitude and thus small in the fluid interior (where formulae to connect the two exist). Near horizontal boundaries, the differences become first order, and so more severe. Existing formulae for quasi-Stokes velocities and streamfunction also break down here. The layer in which the largest differences between the two mean densities occur is the vertical excursion of a mean isopycnal across a deformation radius, at most about 20 m thick. Most climate models would have difficulty in resolving such a layer. It is shown that extant parameterisations appear to reproduce the Eulerian, and not modified mean, density field and so do not yield a narrow layer at surface and floor either. Both these features make the quasi-Stokes streamfunction appear to be non-zero right up to rigid boundaries, so that we must query what are the relevant surface and floor quasi-Stokes streamfunction conditions, and what are their effects on the density fields. To answer this, a variety of eddy parameterisations is employed for a channel problem, and the time-mean density is compared with that from an eddy-resolving calculation. The parameterisations were only successful if the vertical component of the quasi-Stokes velocity vanished at top and bottom as in current practice, but all were almost equally successful given proper tuning. One parameterisation, based on linear instability theory, is extended to a global geometry. In low and mid-latitudes, because the predominant orientation of the instability wavevector is north-south, the main quasi-Stokes flow is east-west, only becoming the more traditional north-south at higher latitudes.

1. Introduction

The ocean component of climate models is necessarily of coarse resolution, and it does not possess eddies. This paper discusses various aspects of the issue of representing eddies in such models. First, it discusses the extant parameterisations (Section 2). Section 3 examines some hitherto unappreciated details, with reference to the 'modified mean density' concept; it is shown that the difference between this density and the Eulerian mean is largest at surface and floor. The region that this difference occupies is too shallow to be resolved by coarse models, so that eddy effects occur in what appears to such models as a delta-function. Current parameterisations do not generate solutions which include these differences, even with adequate resolution. Some eddy-resolving channel experiments are used in Section 5 to explore whether the relevant boundary condition is that of no eddy flux (formally correct beyond the thin layer) or not; the conclusion is that the usual boundary condition must be employed. Last, a parameterisation based on linear instability theory (introduced in Section 4) is extended in Section 6 from channel simulations to the global domain and is briefly compared with the *Gent and McWilliams* (1990) parameterisation. The directionality of linear instability at low and mid-latitudes is such that the 'bolus' fluxes are oriented east-west, rather than north-south, at these latitudes, because the beta effect constrains maximal instability to a north-south orientation (which yields east-west fluxes).

2. Background

A variety of schemes has been suggested to include eddy effects in coarse-resolution ocean models. These schemes divide into two categories. The first, which we shall be examining here, involves adding terms to represent the additional thickness flux by baroclinic eddies (*Gent and McWilliams*, 1990; *Greatbatch and Lamb*, 1990; *Gent et al.*, 1995; *Visbeck et al.*, 1997; *Treguier* *et al.*, 1997; *Killworth*, 1997, 1998; *Greatbatch*, 1998). The second (Neptune) involves a representation of the statistical properties of eddies on the mean flow (*Eby and Holloway*, 1994; *Merryfield and Holloway*, 1997), and will not be discussed in detail here. Methods to represent propagating features (e.g. Agulhas rings) do not seem yet to be available.

Eddy parameterisations have been designed with various criteria in mind by different authors. Initial requirements include mimicking both baroclinic instability – "reducing APE", in some sense, and barotropic instability – "reducing lateral shear", as well as the Neptune approach of making the ocean resemble a rectified eddying ocean. All these are quasi-steady in nature and ignore the fact that instability generates variability on weeks to decades through nonlinear interactions. I am not aware of any parameterisation which attempts to put temporal variability into the ocean (by stochastic forcing, perhaps), so that coupled ocean-atmosphere models, even with parameterisations, are unlikely to possess a realistic degree of self-induced oceanic variability.

Two other pragmatic requirements are that parameterisations should not induce erroneous ocean behaviour (e.g. breaking conservation laws such as momentum), and should ensure that the numerics of the models behave (the original concept behind eddy diffusivity, of course).

The manner in which a parameterisation is couched depends on what belief structure about eddy behaviour is used. In the literature already are suggestions for thickness smoothing, potential vorticity (q) conservation, energy loss, energy conservation, and q smoothing. These effects are usually placed in the tracer equations, but it is possible to include effects in momentum equations also. There are potential difficulties near horizontal surfaces, discussed later.

2.1 Formulations

Most authors seek an "equivalent to the bolus velocity", namely some

$$(u^+, v^+, w^+)$$

which is fully three-dimensional and non-divergent, such that $\bar{\mathbf{u}}+\mathbf{u}^+$ advects (a form of) the density adiabatically – i.e. build in our *belief* that density (or neutral density) effects are adiabatic. This is usually represented as a 2-dimensional streamfunction

$$\Psi = (\psi_1, \psi_2)$$

such that

$$u^+ = \psi_{1z},\ v^+ = \psi_{2z},\ w^+ = -\nabla_H \cdot \Psi.$$

The most logical approach to date is the transient-residual-mean (TRM) theory introduced by *McDougall* (1998, and earlier references therein); *McDougall and McIntosh* (2001, hereafter MM) give more detail on the same material. Another, highly related, approach is to use density-weighted averaging (cf. *Greatbatch*, submitted ms; *de Szoeke and Bennett*, 1993). The TRM theory applies to low-pass temporally averaged quantities and deduces a quasi-Stokes velocity $\mathbf{u}^+$ which is related, but not identical, to the bolus velocity. (The two are not identical because the background mean flow involves averages on two different surfaces, though they are frequently similar.) Formulae have been derived for small perturbations by *McDougall* (1998) and MM, involving only averages at constant depth. The quasi-Stokes vector streamfunction is given to second order in amplitude by

$$\Psi = -\frac{\overline{\mathbf{u}'_H \rho'}}{\bar{\rho}_z} + \frac{\mathbf{u}_{Hz}}{\bar{\rho}_z}\left(\frac{\bar{\phi}}{\bar{\rho}_z}\right),$$

where the suffix H denotes the horizontal component, and $\phi = (1/2)\overline{\rho'^2}$. The vertical derivative of Ψ is the horizontal component of $\mathbf{u}^+$. The second term is usually small compared with the first and is neglected henceforth.

Since eddying motions are believed to conserve density, this implies that the definition of density must be modified. *McDougall* (1998) shows that rather than using the Eulerian mean density $\bar{\rho}$ at a (vertical) point (EMD for short), one should interpret density as being the inversion of the mean depth of a given density (termed the 'modified mean density' $\tilde{\rho}$, or MMD for short). The difference between these two fields $\tilde{\rho}$ and $\bar{\rho}$ is again of second order in small quantities and is thus very small where the TRM theory is formally valid. However, the time derivatives of EMD and MMD differ by O(1) amounts because of the above discussion. The MMD is advected by the (Eulerian) mean flow and by the quasi-Stokes velocity:

$$\tilde{\rho}_t + \nabla \cdot [(\bar{\mathbf{u}} + \mathbf{u}^+)\tilde{\rho}] = 0.$$

We shall see that near horizontal boundaries, the small-amplitude formulae of *McDougall* (1998) and MM to convert EMD to MMD break down. In fact, the two fields differ at first, not second, order in the small quantities. (This is nothing to do with the question of neutrally stable and mixed layers, which are beyond the scope of this paper.)

The earliest parameterisation was a simple pair of diffusivities: $\kappa_H \nabla_H^2 \bar{\rho} + \kappa_V \bar{\rho}_{zz}$. This suffers from the well-known Veronis effect, in that fluid within a density class is not conserved. The now classic GM parameterisation (*Gent and McWilliams*, 1990) takes

$$\Psi = -\kappa \frac{\nabla_H \bar{\rho}}{\bar{\rho}_z}$$

though the density is arguably $\tilde{\rho}$ and not $\bar{\rho}$. This both conserves layer thickness (which, note, is an integral, not a conserved quantity) and "smooths" them. It fits conveniently into the isopycnic mixing tensor formalism of *Solomon* (1981) and *Redi* (1982). *Visbeck et al.* (1997) give a variant with horizontally varying diffusion coefficient based on baroclinic instability. *Killworth* (1997) uses an approximate solution to (single wave) baroclinic instability, obtaining a three-dimensionally varying diffusion coefficient, which vanishes for stable flow. The scheme mixes q in *one direction*, not downgradient. Thus

$$\mathbf{u}_H^+ = \kappa \mathbf{A} \cdot \frac{\partial}{\partial z}\left(\frac{\nabla_H \bar{\rho}}{\bar{\rho}_z}\right) + \frac{\beta}{f}\kappa \mathbf{A}_2$$

$$\mathbf{A} = \begin{pmatrix} \cos^2\theta & -\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin^2\theta \end{pmatrix}$$

and θ is the angle the instability wavevector makes with the x-axis. The approach is usually written in terms of the quasi-Stokes streamfunction. Many other suggestions appear in the literature, mostly untested (*Marshall*, this volume-MAYBE) shows a different approach, which is tested in simple physical situations).

2.2 Inferences from eddy-resolving simulations

There have been several efforts to use eddy-resolving computations to enlighten choices about eddy parameterisation. To date, these have produced not completely consistent results.

Lee et al. (1997) indicate that q is fluxed in a three-layer channel model, and not layer thickness. *Roberts and Marshall* (2000), however, find in a depth-co-ordinate model that the divergent part of the temperature flux is not well correlated with mean temperature gradient; the equivalent for q is moderately well correlated, as are the eddy-induced transport velocity. *Wilson* (2000), however, in a three-layer channel model with forcing varying downstream, finds varying agreement with both thickness and q flux (thickness having better agreement than q) and large areas in the middle layer in which the fluxes are not correlated with either gradient, suggesting the importance of the rotational component. This latter is emphasised by *Drijfhout and Hazeleger* (2001) in a gyral eddy-resolving model; they show that the zonally averaged northward mean thickness gradients are well correlated with the zonally averaged divergent eddy thickness fluxes, while the equivalent for q is much less well correlated.

Estimates of eddy-induced diffusivities also vary. There are strong suggestions that diffusivity varies vertically (*Treguier* 1999; *Robbins et al.*, 2000) as well as theoretical suggestions of lateral variation (*Visbeck et al.*, 1997; *Killworth*, 1997).

The boundary conditions on surface and floor of the TRM streamfunction, which should apparently be zero values in both locations, also remain unclear. Both *Treguier* (1999) and *Gille and Davis* (1999) estimate the streamfunction, which takes extreme values at both top and bottom of the channels they considered.

There is no strong evidence that any single globally tested parameterisation (a) gives similar fluxes (rotational and divergent?) to observed values from eddy-permitting/resolving models or (b) uniformly improves water masses and tracers. GM has by far the largest suite of tests, and while the evidence is clear that – mostly – temperature/salinity distributions are favourably affected by its use (e.g. *Knutti et al.*, 2000 for a 2.5-dimensional model), the response of other tracers deteriorates (*England and Rahmstorf*, 1999).

This should not be surprising: physical eddies have many effects, and the extant parameterisations are aimed at a small subset of those effects.

3. Some details about TRM streamfunctions

3.1 Differences between EMD and MMD

The TRM approach is to work from density coordinates to locate what "density" variable *is* conserved by a flow consisting of a mean ($\bar{\mathbf{u}}$) plus a quasi-Stokes velocity ($\mathbf{u}^+$). *McDougall* (1998) shows that this density (the modified mean density) is the inversion of the average height of a density surface,

$$\bar{z}(x, y, \rho, t) \rightarrow \tilde{\rho}(x, y, z, t)$$

for some averaging operator. Then

$$\tilde{\rho}_t + \nabla \cdot \left\{(\bar{\mathbf{u}} + \mathbf{u}^+)\tilde{\rho}\right\} = 0.$$

MM show that for small amplitude variability, and to second order accuracy,

$$\tilde{\rho} = \bar{\rho} + \hat{\rho} = \bar{\rho} + \mathrm{O}(\alpha^2)$$

$$\hat{\rho} = -\left(\frac{\bar{\phi}}{\bar{\rho}_z}\right)_z, \; \bar{\phi} = \frac{1}{2}\left(\overline{\rho'^2}\right)$$

$$\Psi = -\frac{\overline{\mathbf{u}'_H \rho'}}{\bar{\rho}_z} + \frac{\mathbf{u}_{Hz}}{\bar{\rho}_z}\left(\frac{\bar{\phi}}{\bar{\rho}_z}\right)$$

where the second term is usually small and will be neglected here.

These formulae work well *except near the surface and floor*, where there are *first-order* differences between $\bar{\rho}$ and $\tilde{\rho}$. These differences are produced by the advection of fluid laterally: any fluid of a light density which is ever present at some (x, y) has an entry in the modified mean density. The differences occur over a distance which is first-order in perturbation amplitude. Fig. 1 shows the near-surface behaviour for a flow whose density is given at some horizontal location by

$$z - z_0 = F(\rho - \rho_0) + \alpha G(\rho - \rho_0, t)$$

$$\bar{G} = 0.$$

(The solution is for $G(t) = \sin t$.) The behaviour is caused by time-averaging near surface or floor only being valid when the resulting depth z is within the fluid.

The length scale in Fig. 1 is proportional to eddy amplitude. For linear theory, it appears as a delta-function boundary layer. When the eddies have finite amplitude, the vertical length scale over which the two densities differ noticeably is of order

$$z' \sim \rho'/\bar{\rho}_z \sim a\,|\nabla_H \bar{\rho}/\bar{\rho}_z|$$

which is the typical vertical excursion made when moving a short horizontal distance (a) along a mean isopycnal which moves significantly vertically only on the gyre scale $(L >> a)$. This depth is small for ocean, though not for the atmosphere. Even with fairly optimistic estimates, it is hard to produce a vertical scale much larger than 20 m. So *the distance over which the MMD and EMD differ significantly is not resolved in most climate models*, being concentrated in the last grid point (save in regions such as the Antarctic Circumpolar Current). Thus the near-boundary differences between the two mean densities will probably appear to climate models as single grid-point effects, i.e. delta functions. *McIntosh and McDougall* (1996) show diagnostics from FRAM which support this.

No parameterisation of which I am aware succeeds in reproducing the MMD even given adequate vertical resolution. Fig. 2 shows channel model results for a 4-year and along-channel average of an eddy-resolving calculation, with surface relaxation and parameters designed to permit the lighter near-surface and denser near-bottom fluid present in the MMD to be resolved by the 10-m grid spacing (the relevant depth being 50-60 m). Also shown is a two-dimensional calculation using GM (other parameterisations will give similar poor results). It is clear that the "pushing forward" of warm isopycnals present in the MMD is not present. Differences are very small at the lower boundary because eddy amplitudes were small there also. The presence of

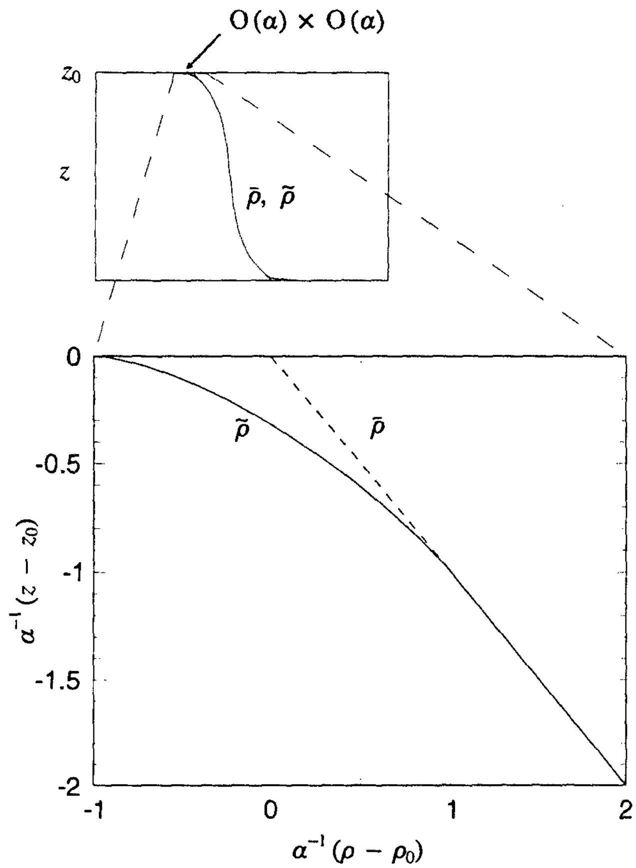

Figure 1. The differences between Eulerian mean and modified density. The upper diagram shows that the densities are very close to each other in the fluid interior (differing by O(α^2), where α is the small amplitude of the fluctuations). In a zone of size α near surface and floor, the two densities differ by a much larger amount, O(α), as indicated in the exploded lower view (which is actually the exact solution for sinusoidal time variation and uniform interior density gradient).

a mixed layer (not treated here) makes no difference to this argument, since it merely moves the region where EMD and MMD differ slightly lower (usually to worse resolution).

Because the resolution does not permit the resolution of the layer where MMD and EMD differ, not all the eddy fluxes can be represented. Fig. 3 shows the the physical near-surface situation contrasted with what is represented in a coarse-resolution model. At the least density which the model is capable of representing, the quasi-Stokes streamfunction is non-zero (its value, correctly, representing the lateral eddy flux within the unresolved layer, which thus appears as a delta-function).

An argument can be made that one could relax the condition of $\psi_1 = \psi_2 = 0$ on surface and floor since the model being run (a) cannot distinguish the EMD and

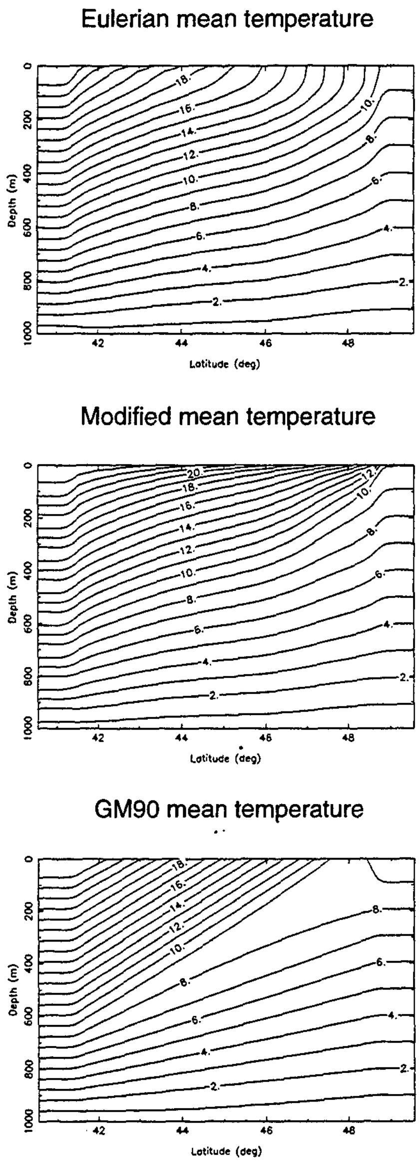

Figure 2. The Eulerian and modified mean density for a 4-year and along-channel average of an eddy-permitting channel model discussed in the text. (The average over the previous 4 year period is almost identical.) The problem was chosen to provide a larger vertical range over which the EMD and MMD differ than would hold for the real ocean, so that the vertical resolution (10 m) was adequate. Also shown is a typical two-dimensional parameterisation steady-state result, in this case following *Gent and McWilliams* (1990), using an eddy diffusion of 2000 $m^2\ s^{-1}$. While the latter does not reproduce the EMD particularly well (true for a wide range of diffusivities), it does not reproduce the MMD at all where this differs from the EMD. This appears to hold for most extant parameterisations.

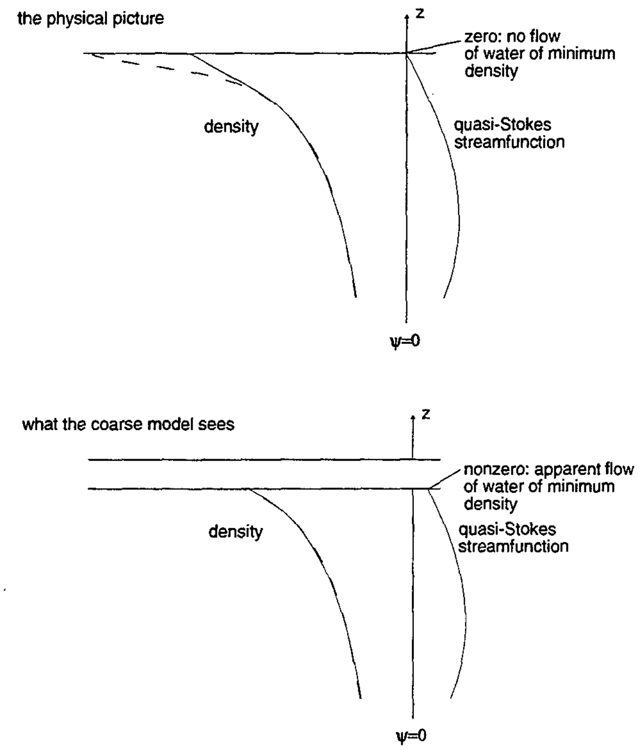

Figure 3. Misrepresentation of quasi-Stokes fluxes when the shallow layer is not resolved by the model

the MMD and (b) does not resolve the missing layer. This is investigated below.

3.2 Mass and available potential energy

The differences between the two densities have two important effects. The first is directly concerned with the interpretation of mean density. It is straightforward to see that the low-pass time filtered net mass in a water column, which is a uniquely defined value, is the same whether EMD or MMD is used:

$$\overline{\int \rho\, dz} = \int \bar{\rho}\, dz \quad \text{(averaging at constant depth)}$$

$$= \overline{\int \rho z_\rho\, d\rho} = \int \rho \bar{z}_\rho\, d\rho = \int \tilde{\rho}\, dz$$

$$\text{(averaging at constant density).}$$

(As Fig. 1 suggests, $\tilde{\rho}$ is lighter at the surface, but the shortfall is made up at the floor.)

The same does *not* hold for potential energy, because of the noncommutative averaging operators on products of quantities. For small amplitude, the differences between EMD and MMD potential energies are $O(\alpha^2)$, and occur due to $O(\alpha^2)$ differences in the interior over a depth range of order unity, and $O(\alpha)$ differences over $O(\alpha)$ depth ranges.

It is shown in *Killworth* (2001) that

$$\Delta PE = \int_{-H}^{0} z(\tilde{\rho} - \bar{\rho})dz < 0.$$

The difference between the two PE expressions lies in the variability, fundamentally a part of the MMD. It involves an integral in density space of the mean square depth fluctuations; the proof is straightforward, either from the *McDougall* (1998) formulae or by direct evaluation, and is not given here.

Thus *the low-pass filtered potential energy of a fluid column (a uniquely defined quantity) is only correctly evaluated using EMD, and is consistently underestimated using the MMD*; correction terms can be derived, and involve knowledge of the variability. This implies that energetics cannot consistently be produced for coarse models including parameterisations.

4. Creating a parameterisation

The *Gent and McWilliams* (1990) parameterisation has the virtue of simplicity and elegance, although to function globally various modifications have to be made, typically those of 'tapering' neutral density slopes when they become too large. Other parameterisations are usually less elegant, and so need more modifications. The depth-co-ordinate version of *Killworth* (1997) is briefly discussed here before comparisons are made in channel and global geometries.

The parameterisation uses linear theory, following *Robinson and McWilliams* (1974). Linear theory is not always a good predictor of the behaviour of a nonlinear eddying system, as *Edmon et al.* 1980) demonstrate, but it makes an indicative starting point (and holds in the same parameter range as the *McDougall* (1998) formulae). The system is assumed to be slowly varying in the horizontal, compared with the local deformation radius. The equations for small perturbations varying as $\exp ik(x\cos\theta + y\sin\theta - ct)$ are

$$(\tilde{u} - c)\left\{\left(\frac{f^2 p_z}{N^2}\right)_z - k^2 p\right\} + \tilde{q}_y p = 0$$

$$\tilde{q}_y = \beta\cos\theta - \left(\frac{f^2}{N^2}\tilde{u}_z\right)_z \ (q \text{ in direction } \theta)$$

$$\tilde{u} = u\cos\theta + v\sin\theta \ (\text{velocity in direction } \theta)$$

In the case of a channel geometry, θ is identically zero.

All quantities can be expressed in terms of the pressure perturbation, for example:

$$\overline{v'\rho'} = -\frac{k\cos\theta}{2f\rho_0 g}\mathrm{Re}(ipp_z^*);\ \psi_2 = -\frac{k\cos\theta}{2f\rho_0^2 N^2}\mathrm{Re}(ipp_z^*)$$

$$\text{diffusivity } \kappa = \frac{kc_i}{2f^2\rho_0^2}\left|\frac{p}{\tilde{u} - c}\right|^2$$

$$u^+ = -\frac{\kappa\sin\theta\,\tilde{q}_y}{f};\ v^+ = \frac{\kappa\cos\theta\,\tilde{q}_y}{f}.$$

Thus to estimate Ψ, all that is needed is an *approximate* solution to the instability problem. The deformation radius is estimated from

$$C = \frac{1}{\pi}\int_{-H}^{0} N(z)\,dz, a = C/f$$

and wavenumber from the *Eady* (1949) result

$$k = 0.51/a.$$

The angle θ is estimated from a crude solution maximising the growth rate based on standard deviations of u and v over depth. As shown by *Gill et al.* (1974), θ is small for east-west flow as in the channel case, but is near $\pi/2$ if u is small and v nonzero except in high latitudes where β is small. The approximate vertical structure of κ and p is found by two iterations of an approximate (small wavenumber) solution of the problem, and κ is scaled by Aac_i. Here A is an O(1) constant: 3 is found to be optimal. The use of c_i, the imaginary part of the phase speed, ensures that there is no mixing where flow is stable, though there is always instability when v is non-zero, as *Pedlosky* (1987) shows.

The resulting streamfunction possesses two terms. The first is $-\kappa$ times the slope of the isopycnals normal to the angle of instability, which has similarities to the GM formulation. The second, extra term, has no such easy interpretation. Here the diffusivity κ varies three-dimensionally.

5. Channel model comparisons

The discussion in Section 3 suggests that the surface and floor conditions on the TRM in coarse resolution models are not obvious (Fig. 2 being a good example). This section examines solutions to two-dimensional emulations of the three-dimensional channel model of *Killworth* (1998), using a variety of formulations to represent the eddy terms, specifically to examine the boundary condition question.

The model covered a longitude range of 2.6°, a latitude range of 5.2°, centred on 30°N, and a shallow depth of 300 m. The grid spacings were 0.02° east-west, 0.018° north-south and 20 m vertically, with viscosities 50 m^2 s^{-1}(horizontal) and 5 × 10^{-4} m^2 s^{-1}vertically and diffusivities 10 m^2 s^{-1}horizontally and 10^{-4} vertically. Starting from a narrow temperature front with uniform salinity, relaxation towards the initial temperature values in bands at north and south of the channel provided a source of potential energy. This method has the advantage that there are no regions of unstable or neutral stratification, thus avoiding difficulties about parameterisations in such regions. Averages were computed over time and longitude over 7.25 years between days 300 and 2950.

Two-dimensional (latitude-depth) simulations were then run using a variety of two-dimensional parameterisations on a Cartesian grid, and the 4000-day computations (steady in almost all cases) compared with the averages from the three-dimensional run. Comparisons were made with the temperature field as a function of y (north) and z, and with the baroclinic u velocity.[1] The comparisons are not ideal. Like other published work, they are of Eulerian means only, and over a period probably an order of magnitude too short for a good statistical comparison. (However, the fields in Fig. 3 were visually unaltered by averaging over another period of similar length, so the statistics may be better than we suggest.)

Comparisons were made both visually and using a stringent measure of explained variance due to *Visbeck et al.* (1997). In each case, any free parameters in the parameterisation were adjusted to maximise the agreement with the eddy-resolving average. No parameterisation reproduced the 'pushing forward' of isopycnals in the MMD, so that direct comparisons with it are not useful.

Fig. 4 shows the three-dimensional time- and along-channel-averaged solution. To provide a yardstick for the various parameterisations, Fig. 5 shows the two-dimensional temperature field using only advection by the actual (two-dimensional) velocity fields plus a horizontal diffusivity of 200 m^2 s^{-1}, which clearly gives results very close to the three-dimensional results.

The other parameterisations used were (in order of appearance in Figs. 6-10) the following:

GM90 (*Gent and McWilliams*, 1990, which has a constant diffusivity); Fig. 6

K97 (more properly, the depth co-ordinate version of *Killworth*, 1997, discussed earlier, which computes a variable diffusivity); Fig. 7

GMs (*Gent and McWilliams*, 1990, but with the streamfunction non-zero at the surface); Fig. 8

Ks (*Killworth* 1997, adapted as discussed below); Fig.9

VP (computing $(\overline{v'\rho'})_y$ directly from small-amplitude formulae, also discussed below); Fig. 10

[1]As discussed by *Killworth* (1998), the two-dimensional runs have no depth-averaged u field, so that only the baroclinic u can be compared. The barotropic u field, as noted by *Killworth*, plays a not inconsiderable role in the dynamics.

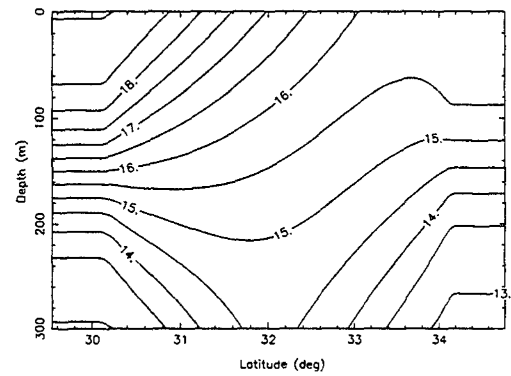

Figure 4. Contours of temperature (°C; contour interval 0.5°C) for the time- and along-channel-averaged three-dimensional eddy-resolving calculation.

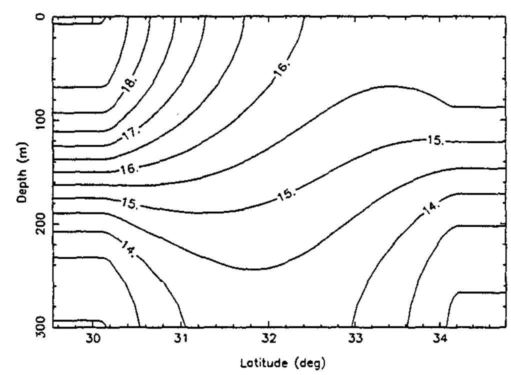

Figure 5. Two-dimensional simulation of Fig. 4, using a horizontal diffusivity of 200 m^2 s^{-1}

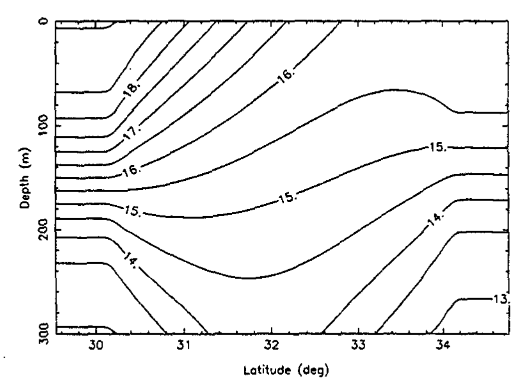

Figure 6. As Fig. 4, but using the *Gent and McWilliams* (1990) parameterisation, with $\kappa = 160$ m^2 s^{-1}

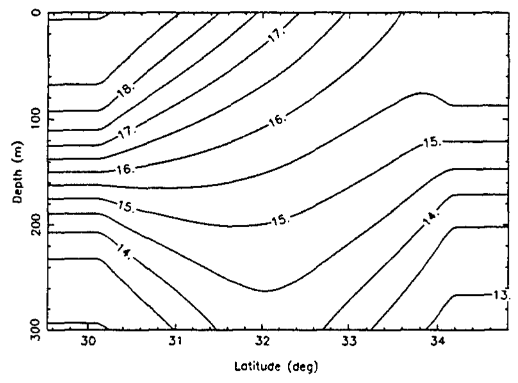

Figure 7. As Fig. 4, but using the Killworth (1997) parameterisation with $A = 3$.

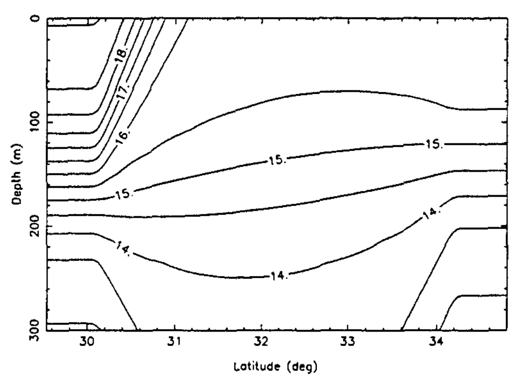

Figure 8. As Fib. 4, but using the Gent and McWilliams parameterisation modified so that the streamfunction does not vanish at surface or floor, with $\kappa = 1200$ m^2 s^{-1}

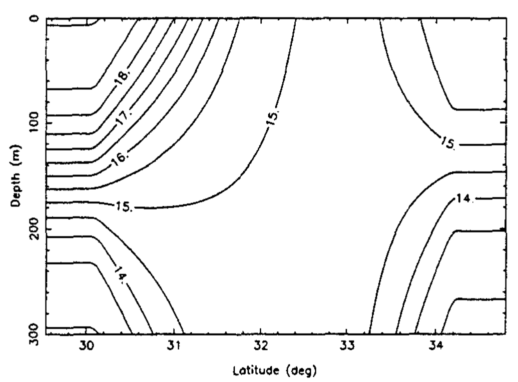

Figure 9. As Fig. 4, but using the Killworth (1997) parameterisation, modified so that the streamfunction does not vanish at surface or floor, with $A = 5$

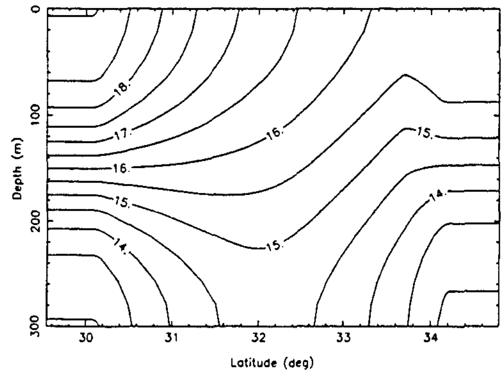

Figure 10. As Fig. 4, but using a direct parameterisation of $(\overline{v'\rho'})_y$ directly from linear theory, with $A = 3$

The GM parameterisation is handled as in the MOM3 code (*Pacanowski and Griffies,* 1999), with the TRM streamfunction dropping to zero over the last grid point. The diffusivity κ is taken as a constant. The K97 parameterisation is as discussed earlier. The third parameterisation, GMs, attempts to emulate a nonzero value of streamfunction at surface and floor. This is not an easy task numerically, since many apparently straightforward approaches generated numerical instabilities. These included extrapolation of either the isopycnic slope or the streamfunction to the boundary, and computation of boundary values using one-sided interpolation formulae. A slightly unsatisfactory approach which set the streamfunction at surface (floor) to the values immediately below (above) was eventually used; the disadvantage being that the v^+ field vanished in the top and bottom grid points. The fourth parameterisation, Ks, does the same thing for K97.

The last parameterisation, for the EMD, directly evaluates $(\overline{v'\rho'})_y$ (very similar answers are found for $(\overline{v'\rho'})_y + (\overline{w'\rho'})_z$) directly from small-amplitude theory, again using *Killworth's* (1997) scalings. Neither calculation requires boundary conditions since the normal velocity must vanish at boundaries. Direct attempts to parameterise the flux divergence usually suffer from Veronis effects (*Veronis,* 1975); however, this approach does not, since the terms have the same conservation properties (for the flux terms) as the original system.

The most accurate version of the GM90 parameterisation for this problem has a κ of 160 m^2 s^{-1}, a little lower than that cited in *Killworth* (1998). The results for the GM90 (Fig. 6) are very similar to those of pure diffusion (5), although slightly less accurate in the u field. The similarity is surprising since the GM90 includes the strong northward (southward) advection near the surface (floor) which is not present in the simple diffusive case.

The most accurate version of the K97 parameterisation (Fig. 7) has $A = 3$, as used in *Killworth* (1998) for the same problem. As Fig. 7 shows, this parameterisation is the only one to produce the 'doming' of the 15.5° isotherm near the northern boundary with any accuracy.

If w^+ is not required to vanish at surface and floor, then for this geometry the parameter values used hitherto are insufficient to reproduce the three-dimensional solution. This is because the high northward advection near-surface is now lacking. For the GM90 parameterisation (Fig. 8), κ needed to be increased an order of magnitude (to 1200 m^2 s^{-1}) in order to reproduce an approximation to the three-dimensional fields. Although the temperature field looks reasonable, the corresponding velocity is poorly reproduced, due to the strong surface front near the southern boundary. A similar finding holds for the K97 parameterisation (Fig. 9). Thus permitting non-zero w^+ at surface and floor has not achieved a higher accuracy than maintaining zero w^+, for this problem and choice of parameterisations.

The final parameterisation (VP) does not use the (v^+, w^+) formulation, but simply inserts a parameterisation for mixing directly. The results (Fig. 10) again show an accurate representation of the three-dimensional result.

In terms, then, of reproducing the *Eulerian* mean density, most schemes were successful. The K97 and VP were only marginally superior to the others, and schemes which permitted nonzero quasi-Stokes streamfunctions at the surface were quite inferior.

6. Global parameterisations

Although various theoretical suggestions for parameterisations have been made, relatively few have been tested in global models. GM has been shown to be robust (provided that tapering arrangements are included to prevent poor behaviour in weakly stratified environments). The K97 parameterisation is here extended to work globally, within the MOM3 code *(Pacanowski and Griffies,* 1999). This represents a TRM streamfunction as an extra term within the 3×3 isopycnic mixing tensor, as

$$\begin{pmatrix} A_I & 0 & A_I S_x + \psi_1 \\ 0 & A_I & A_I S_y + \psi_2 \\ A_I S_x - \psi_1 & A_I S_y - \psi_2 & A_I S^2 + A_D \end{pmatrix} \begin{pmatrix} \rho_x \\ \rho_y \\ \rho_z \end{pmatrix}$$

where S_x, S_y are the isopycnal slopes in the x and y directions, A_I is the isopycnal diffusion, and A_D the diapycnal diffusion.

First, there is also an element of tapering. In the expressions for ψ_1 and ψ_2 (not shown here) the first term, as noted earlier, includes an expression which is the product of the diffusivity and the isopycnal slope normal to the orientation of the fastest growing wavenumber. This slope is tapered in precisely the same way as in isopycnal mixing. Second, because the K97 formulation is essentially quadratic in shear (κ increases with shear, and operates on the shear) rather than linear as in GM, an abrupt start, e.g. from observed temperature and salinity, could lead to instabilities. The diffusivity – and hence streamfunctions – within a column are rescaled if necessary so that $\kappa < \kappa_{max} = 10^4$m^2 s^{-1}.

The role of baroclinic instability near the equator is unclear. While GM does not depend on position, any scheme involving q mixing must make choices near equator which reflect (a) that the role of baroclinic instability is less near the equator than at mid-latitude and (b) that it is the east-west density gradient which is predominantly creating flows, and so should be preferentially decreased by eddies. In addition, preliminary results analysing OCCAM output (*de Vries and Drijfhout,* personal communication) suggest that the TRM streamfunction is much larger near the equator than a simple constant-diffusion GM would predict.

In any event, an engineering 'fix' is required. At present, the Coriolis parameter $\hat{f}$ is taken as the maximum of f and some f_{min} (N. Hemisphere sign convention), and the deformation radius is defined from the mid-latitude and equatorial values by

$$a_{mid} = C/\left|\hat{f}\right|; a_{eq} = \sqrt{(C/2\beta)};$$

$$a = \min(a_{mid}, a_{eq}); k = 0.51/a_{mid}$$

which both avoids large wavenumbers and permits the approximate solution to continue to function reasonably.

The last modification is that velocity shears are consistently used in the calculation instead of density gradients, since thermal wind fails near the Equator. Near the surface these include the Ekman contribution. This is removed in the surface layer (the model has no mixed layer) by extrapolating the effective surface velocity from the next two depths.

The above changes have been implemented in MOM3, and run in a 2° × 2° near-global configuration (77°S – 77°N) for one year only, since various features of the parameterisation remain under experiment. Monthly windstress and surface forcing were employed using standard MOM3 options. Typical parameter values were used, including an isopycnal diffusivity of 10^3 m^2 s^{-1}. A parallel run used the GM formulation, which is an option in MOM3, with $\kappa = 10^3$ m^2 s^{-1}. This failed after 3 months with erroneously high velocities in the Arctic. Results from the last snapshot files are shown from both runs. Normal diagnostics such as overturning streamfunction are not useful with such short runs, and for reasons of space only one elementary diagnostic is given here.

Figures 11 and 12 show the near-surface (level 1) horizontal TRM velocities superimposed on temperature contours for K97 and GM at the second level (depth 37.5 m) of the model, at the respective ends of their runs. Interestingly, neither set of TRM velocities are particularly small compared with the mean flow. The K97 TRM flow is predominantly east-west at latitudes less than about 40°. This is caused by the north-south orientation of the fastest growth rate, at least with the approximation used here. Another approximation, casting the continuous flow onto a 2-layer system and maximising growth rate over wavenumber and angle, followed by using this in the vertical iteration scheme used normally in K97, gives similar answers. It is possible that both approximations miss other, stronger growth rates at higher wavenumber. *Gill et al.* (1974) give some examples, but the indication from their work and from *Pedlosky's* (1987) book is that the orientation is expected to be close to north-south under most circumstances, as Fig. 11 suggests. Thus the restriction of the TRM flow to represent q mixing along a single orientation only yields TRM flows which are in the east-west direction in midlatitudes (despite the fact that density gradients are for the most part much stronger north-south than east-west in ocean data). Only in the subpolar areas is the β-effect reduced sufficiently that north-south TRM flows appear, e.g. in the Antarctic Circumpolar Current. However, the equatorial TRM flows in Fig. 11 are smooth and well-behaved, in contrast to the GM flows in Fig. 12 which are rather noisy. In the subtropics, both K97 and GM have weak TRM flows. However, in the subpolar region the GM TRM flows are more uniform and slightly stronger poleward than their K97 counterparts.

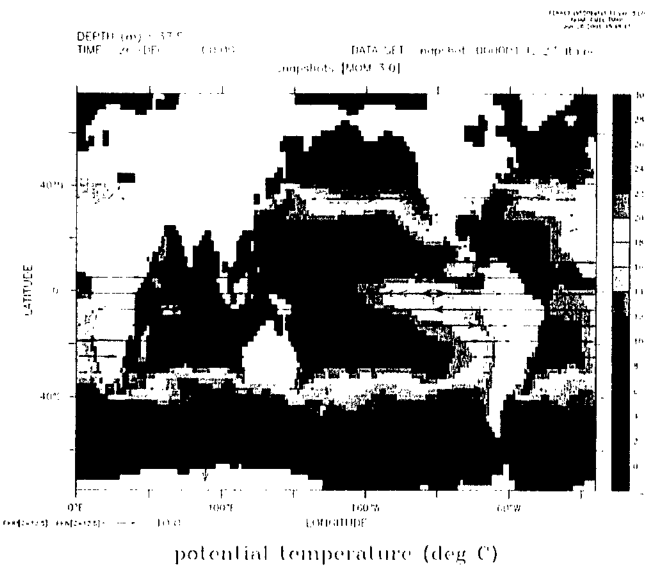

Figure 11. Global K97 results for near-surface temperature and horizontal TRM fluxes, after 1 year's integration

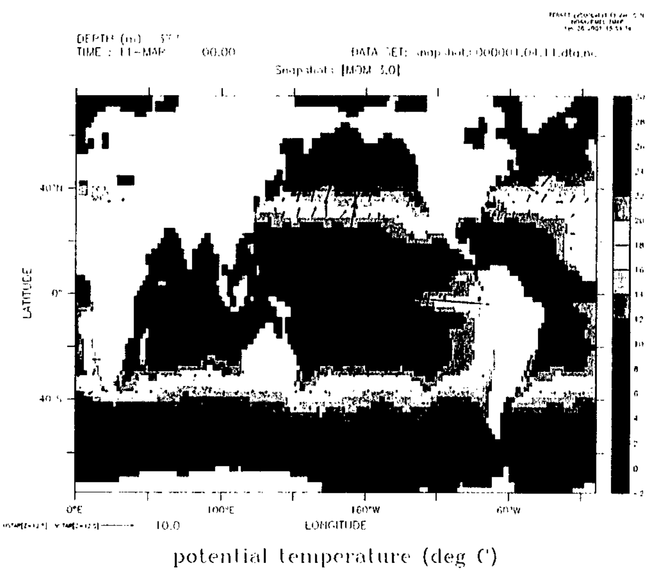

Figure 12. GM equivalent, after 3 months' integration

7. Discussion

The eddy parameterisation issue is far from resolved for many reasons. Even if the belief structure is adopted that mixing is done by eddies, it is unclear what should be mixed. The role of divergent and rotational fluxes remains unclear. Coarse models do not usually possess the vertical resolution to distinguish Eulerian and MM density near the surface and floor, save in weakly stratified regions.

Nonetheless, simple, clean parameterisations such as GM seem to improve much (but not all) of a coarse model response. There are indications that mixing coefficients vary spatially, but it is unclear whether this is O(1) in importance; recall that the channel simulations showed almost indistinguishable results between GM, with a constant diffusivity, and K97, where the diffusivity varied by two orders of magnitude. Similarly, more complicated parameterisation schemes appear to function as well as (sometimes better, sometimes not) simple schemes. Again it is unclear how important the simple-complex distinction is. Certainly it is not proven that any complicated scheme performs better globally than a simple one.

In the near future, careful intercomparisons of different schemes against trusted, statistically reliable three-dimensional eddy-resolving calculations will be required. This needs the community to invest in fine-resolution closed-gyre forced and free calculations run to some form of equilibrium to use as the testbeds for parameterisations. Some of these need to be explicitly time-varying, e.g. on seasonal scales. (Should seasonal variation be included within, or without, the averaging interval for eddy fluxes when applied, say, to annual mean coarse resolution modelling?) The community should also be encouraged to seek other ways of casting the parameterisation problem (e.g., the approach by Marshall in this volume).

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References

de Szoeke, R.A., and A.F. Bennett, Microstructure fluxes across density surfaces. *J. Phys. Oceanogr.*, *23*, 2254-2264, 1993.

Drijfhout, S.S., and W. Hazeleger, Eddy mixing of potential vorticity versus thickness in an isopycnic ocean model". *J. Phys. Oceanogr. 31,* 481-505, 2001.

Eady, E.T., Long waves and cyclone waves. *Tellus, 1*, 33-52, 1949

Eby, M., and G. Holloway, Sensitivity of a large-scale ocean model to a parameterisation of topographic stress. *J. Phys. Oceanogr.*, *24*, 2577-2588, 1994.

Edmon, H.J., B.J. Hoskins, and M.E. McIntyre, Eliassen-Palm Cross Sections for the Troposphere. *J. Atmos. Sci.*, *37*, 2600-2616, 1980.

England, M.H., and S. Rahmstorf, Sensitivity of Ventilation Rates and Radiocarbon Uptake to Subgrid-Scale Mixing in Ocean Models. *J. Phys. Oceanogr.*, *29*, 2802-2828, 1999.

Gent, P.R., and J.C. McWilliams, Isopycnal mixing in ocean circulation models. *J. Phys. Oceanogr.*, *20*, 150-155, 1990.

Gent, P.R., J. Willebrand, T.J. McDougall, and J.C. McWilliams, Parameterizing Eddy-Induced Transports in Ocean Circulation Models. *J. Phys. Oceanogr.*, *25*, 463-474, 1995.

Gill, A.E., J.S.A. Green, and A.J. Simmons, Energy partition in the large-scale ocean circulation and the production of mid-ocean eddies. *Deep-Sea Res.*, *21*, 499-528, 1974.

Gille, S.T., and R.E. Davis, The influence of mesoscale eddies on coarsely resolved density: An examination of subgrid-scale parameterisation. *J. Phys. Oceanogr.*, *29*, 1109-1123, 1999.

Greatbatch, R.J., Exploring the relationship between eddy-induced transport velocity, vertical momentum transfer, and the isopycnal flux of potential vorticity. *J. Phys. Oceanogr.*, *28*, 422-432, 1998.

Greatbatch, R.J., and K.G. Lamb, On parameterizing vertical mixing of momentum in non-eddy resolving ocean models. *J. Phys. Oceanogr.*, *20*, 1634-1637,

Killworth, P.D., On the parameterisation of eddy transfer. Part I: Theory. *J. Mar. Res.*, *55*, 1171-1197, 1997.

Killworth, P.D., On the parameterisation of eddy transfer. Part II: Tests with a channel model. *J. Mar. Res.*, *56*, 349-374, 1998.

Killworth, P.D., Boundary conditions on quasi-Stokes velocities in parameterizations. *J. Phys. Oceanogr.*, *31*, 1132-1155, 2001.

Knutti, R., T.F. Stocker, and D.G. Wright, The effects of subgrid-scale parameterizations in a zonally averaged ocean model. *J. Phys. Oceanogr.*, *30*, 2738-2752, 2000.

Lee, M.M., D.P. Marshall, and R.G. Williams, On the eddy transfer of tracers: advective or diffusive. *J. Mar. Res.*, *55*, 483-505, 1997.

McDougall, T.J., Three-dimensional residual mean theory. pp. 269-302 in *Ocean Modeling and parameterisation*, eds. E. P. Chassignet and J. Verron. Kluwer, 451 pp., 1998.

McDougall, T.J., and P.C. McIntosh, The temporal-residual-mean velocity Part II: Isopycnal interpretation and the tracer and momentum equations. *J. Phys. Oceanogr.*, *31*, 1222-1246, 2001.

McIntosh, P.C., and T.J. McDougall, Isopycnal averaging and the residual mean circulation. *J. Phys. Oceanogr.*, *26*, 1655-1660, 1996.

Merryfield, W.J., and G. Holloway, Topographic stress parameterisation in a quasi-geostrophic barotropic model. *J. Fluid Mech.*, *341*, 1-18, 1997.

Pacanowski, R.C., and S.M. Griffies, MOM 3.0 manual, Geophysical Fluid Dynamics Laboratory, Princeton, NJ, 680 pp., 1999.

Pedlosky, J., *Geophysical Fluid Dynamics.* 2nd ed. Springer Verlag, 710 pp., 1987.

Redi, M., Oceanic isopycnal mixing by coordinate rotation. *J. Phys. Oceanogr.*, *12*, 1154-1157, 1982.

Roberts, M.J., and D.P. Marshall, On the validity of down-gradient eddy closures in ocean models. *J. Geophys. Res.*, *105*, 28,613-28,628, 2000.

Robbins, P.E., J.F. Price, W.B. Owens, and W.J. Jenkins, The importance of lateral diffusion for the ventilation of the lower thermocline in the sub-tropical North Atlantic. *J. Phys. Oceanogr.*, *30*, 67-89, 2000.

Robinson, A.R., and J.C. McWilliams, The baroclinic instability of the open ocean. *J. Phys. Oceanogr.*, *4*, 281-294, 1974.

Solomon, H., On the representation of isentropic mixing in ocean circulation. *J. Phys. Oceanogr., 1,* 233-234, 1971.

Treguier, A.M., Evaluating eddy mixing coefficients from eddy-resolving ocean models: A case study. *J. Mar. Res., 57,* 89-108. 1999.

Treguier, A.M., I.M. Held, and V.D. Larichev, On the parameterisation of quasi-geostrophic eddies in primitive equation ocean models. *J. Phys. Oceanogr., 27,* 567-580, 1997.

Veronis, G., The role of models in tracer studies. In *Numerical Models of the Ocean Circulation.* Natl. Acad. of Sci., 133-146, 1975.

Visbeck, M., J. Marshall, T. Haine, and M. Spall, On the specification of eddy transfer coefficients in coarse resolution ocean circulation models. *J. Phys. Oceanogr., 27,* 381-402, 1997.

Wilson, C., The role of mesoscale eddies and its representation in numerical models. PhD thesis, University of Liverpool, 128 pp., 2000.

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